# GPS Position Variability Experiment

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## Experiment Scenario

This experiment was intended to establish a familiarity with what the typical position error variability might be, of a newly acquired GPS unit (Garmin eTrex Legend), in scenarios that the author would typically expect to encounter. The experiment used two different locations, each well open to the sky, where one might typically be inclined to enter into and briefly stop, in an attempt to get a better location fix. The first location was along a sidewalk, on a mature residential urban street, with some tall trees, and occasional patches of smaller trees that hang quite low, directly overhead. The second location was in the middle of an open field, on the side of an urban park, with densely packed trees toward the park center, and very tall but less dense trees along the outer edge. Each site location seemed to have directions with potentially good sky views, that looked to be below say 45° elevation, toward the horizon. Contrary to this, the user was always blocking reception from some direction, with the unit at about waist height.

On various days (52 actually) in June, July and August, 2002, each of the two locations was visited according to an ad hoc routine, typically once or twice on a given day, on foot or with the unit mounted on the handlebars of a bicycle, and with some amount of variation in the direction of approach to each site. Any given visit to one location was followed soon after by a visit to the other (a few hundred meters away), such that the sky conditions (weather and satellites) for each location's data set could be considered essentially the same. (The order in which the two locations were visited also changed from time to time.) On any given visit the unit was typically locked and tracking (at least as indicated) for at least one minute, and on arrival, a single location fix was recorded within a few seconds after having stopped. The unit used in this work displays an updated location about once per second. Recording a single location fix is accomplished by pressing a button to store the current location coordinates in memory. A total of 80 fixes were collected for each of the Street and Field locations, over the entire experiment.

## Analytical Bits and Pieces

A typical 2D surface position measurement, or location fix, is displayed as something like 18T 0442129 5026605, when the GPS unit is set to read in the Universal Transverse Mercator (UTM) coordinate system*. The 18T indicates the particular UTM zone in which the fix lies, and the next two numbers indicate the East and North coordinates, respectively, measured in meters with respect to the standard reference grid, for the given UTM zone.

The coordinates of the centroid, $X_C$, $Y_C$, of N location fixes, may be computed using

$$X_C = \frac{\left(\sum_{n=1}^{N} x_n\right)}{N} \qquad Y_C = \frac{\left(\sum_{n=1}^{N} y_n\right)}{N}$$

where $x_n$ and $y_n$ are the UTM East and North coordinates of fix n, respectively. (Note, N=80 in this work.) Figures 1 and 2 illustrate the sets of Street and Field location fixes, respectively, where each set is plotted relative to its centroid. (i.e. each fix, n, is located at $x_n-X_C$, $y_n-Y_C$)

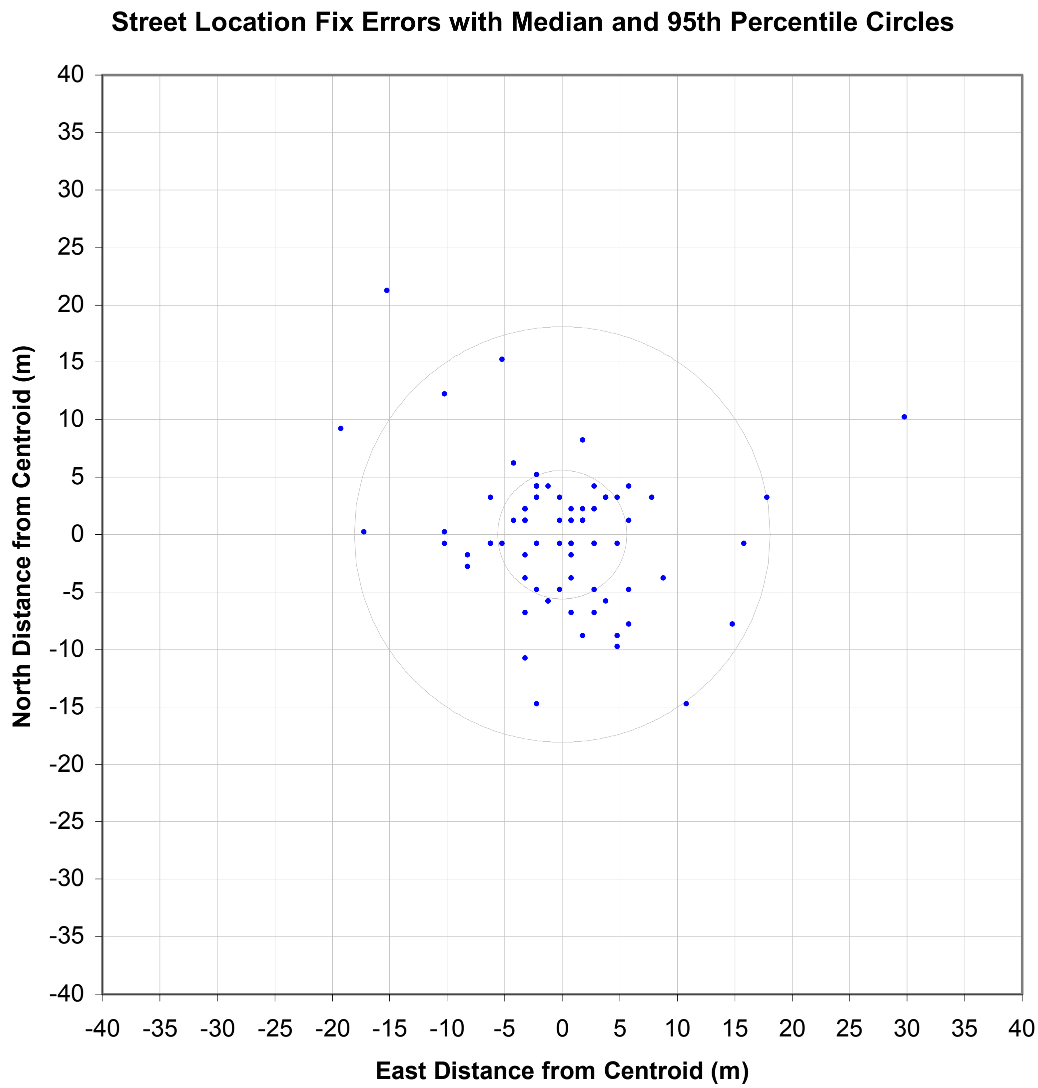


**Figure 1** – Street location fixes, with error probability circles.

The radial distance of a fix from the centroid of a large set of fixes (taken at the same location) is assumed here, and typically in other published works, to be a practical measure of the 2D position error of that location fix. The distance error of fix n, denoted here as $d_n$, is given by

$$d_n = \sqrt{(x_n - X_C)^2 + (y_n - Y_C)^2}$$

The RMS value of the set of radial distances from the centroid is used here, and in other typical published works, to characterize the nominal error of location fixes encountered in a given experiment data set. This value, often denoted as dRMS, is given by

$$dRMS = \sqrt{\frac{\left(\sum_{n=1}^{N} d_n^2\right)}{N}}$$

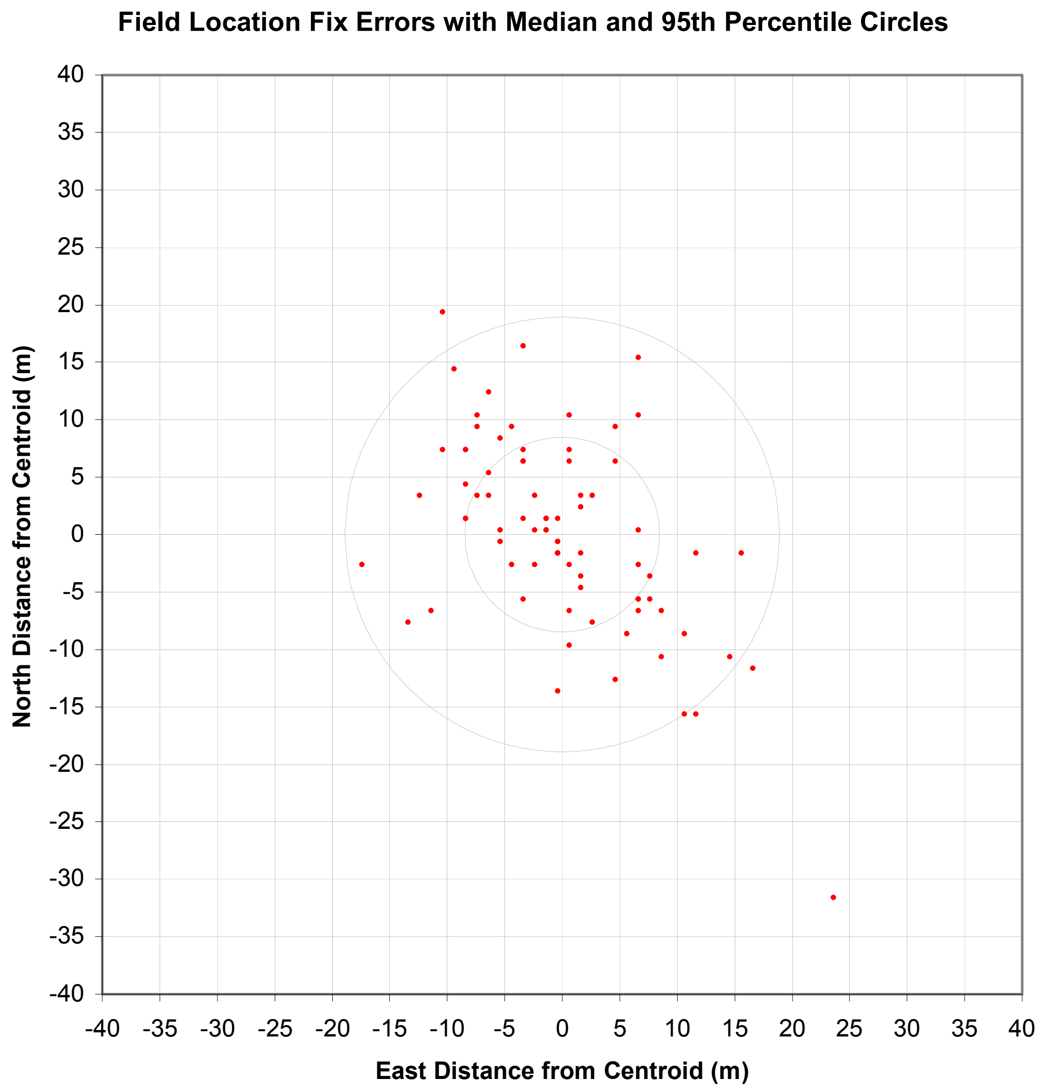


**Figure 2** – Field location fixes, with error probability circles.

## Discussion of the Results

Table 1 contains a list of various computed nominal radial distance errors, for each of the Street and Field datasets, including the dRMS value described above, 2dRMS (2 x dRMS), the median (50th percentile), and the 95th percentile distance. If radial distance errors followed a two-dimensional Normal distribution about the centroid, then one would expect to find about 68.3% of all location fixes with distance errors of less than dRMS, and about 95.4% of fixes with error less than 2dRMS. However, the distribution of these radial distance errors apparently tends more toward a Rayleigh rather than Normal distribution, according to the literature, and 2dRMS should tend to represent an even more conservative (pessimistic) estimate of the 95th percentile error radius (i.e. should encompass even more than 95.4%). This seems to be supported (or at least not contradicted) by the (admittedly statistically limited) distance errors in Table 1.

**Table 1** – Nominal radial distance errors (m) of the Street and Field sets of fixes.

| (meters) | **Street** | **Field** | *Avg* |
|---:|---|---|---|
| **median** | 5.6 | 8.5 | *7.0* |
| **dRMS** | 9.2 | 11.1 | *10.1* |
| **95th percentile** | 18.1 | 18.9 | *18.5* |
| **2dRMS** | 18.3 | 22.2 | *20.3* |

The plots in each of Figures 1 and 2 contain circles, centered on the origin, with radii of the median and 95th percentile distance errors listed in Table 1. It can been seen from the plots, even without considering the numerical statistics, that the street data clearly has a 'tighter' (higher) central distribution (peak) than the field data, with a median of 5.6 m for the street versus 8.5 m for the field, whereas their 95th percentile limits are quite similar, at 18.1 m and 18.9 m, respectively. (It is noted that the GPS unit displayed coordinates only to the nearest meter, and although not visible, a number of fixes are plotted on top of each other in Figures 1 and 2.) These outer region estimates are admittedly still quite rough and sensitive, with so few data points (only 4 beyond 95%).

It is speculated that the apparently significant difference between the central distributions of the two locations might be accounted for by differences in the surroundings near ground level. On the street, the view at lower elevation, say well below 45°, tended to be either clear, or totally blocked by structures, such that when low-elevation satellites were in view, particularly accurate fixes might be acquired. In the field, one could see tall trees in almost any direction, at least at some distance less than say a hundred meters, such that maybe the low-elevation satellite signals tended to be degraded, or at least received more sporadically, resulting in a 'spreading' of the otherwise 'good' fixes that would have been taken with a particularly good satellite configuration. (Note, as mentioned earlier above, a fix taken at one location was followed fairly quickly by a fix at the other location, keeping the satellite configuration similar.)

The above explanation for the 'spreading' would seem to require that the GPS unit has some amount of exponential averaging or estimating taking place, so that when signals are being 'tracked', but drop out for short periods of time, the position estimate only degrades, as opposed to jumping to a new solution that in no way relates to the previous one. If such averaging or estimation is in fact taking place, then this also indicates that how long one remains stationary, before storing the current location in memory, may affect the accuracy of that fix; in other words it might tend to get a little better while you stand there. This idea hasn't been checked with the unit used in this work, but during data collection a conscious effort was made to always wait just a few seconds before storing a fix; not doing so immediately upon stopping, but still fairly promptly, say within 2 to 5 seconds, as one might do on a casual hike or ride.

## Summary

Testing of the performance of GPS units themselves (as opposed to more convoluted testing that includes variability in a typical user's hands) is apparently often done in open spaces, with good visibility of the horizon in all directions, which should potentially allow tracking as many as a dozen satellites at once. (In this work, the unit usually indicated that say four to six satellites were being tracked, although this wasn't observed carefully.) Such studies often acquire tens of thousands of fixes for their statistical analysis, over all hours of the day (time-of-day was not considered in this work). Such studies would seem to allow for proper evaluation and comparison of units, at least under fairly ideal conditions.

The sparse distributions of the Street and Field location fix data in this work are certainly not as 'text-book' as those that the author has encountered in limited study of the literature, however they would seem to be about what was expected, and are in fact believed to be quite representative of what one would encounter in similar practical applications. Simply averaging the two sets of data in Table 1 (ignoring any statistical arguments against do so), we will say that 50% of fixes were within about 7.0 m of the true location, and that 95% of fixes were within about 18.5 m. It is noted that the maximum horizontal position error for the Garmin eTrex Legend seems to be specified to be 15 m, although it is unclear to the author what measurement conditions (usage scenario) one should assume, and whether this number represents say the 95th percentile or 2dRMS error. In any case, considering the limited nature of this experiment, the findings seems to be fairly well in-line with the specification.

*** Reading UTM Coordinates:**

A typical 2D surface position measurement, or location fix, might be displayed as something like 18T 0442129 5026605, in the Universal Transverse Mercator (UTM) coordinate system. The 18T indicates the particular defined UTM zone, a patch on the earth that is 6º 'wide' in longitude and 8º 'high' in latitude, in which the fix lies; zone 18T is defined as 72º to 78º west longitude by 40º to 48º north latitude. The next two numbers indicate the East and North coordinates, respectively, measured in meters, relative to the standard UTM grid references. Specifically, the East coordinate is set to 0500000 meters (500km) at the central meridian line passing 'up' through the centre of the 6º wide zone (75º in this case), and the North coordinate is set to zero at the equator. The above position fix then lies 57.871 km West (500-442.129) of the central zone meridian of 75°, and 5,026.605 km North of the equator.